Transition from regular to chaos from the aspect of nodal domains associated with wave functions

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2次元4次振動子模型を用いて、系のダイナミクスが可積分からカオスまで変化するとき、波動関数の結節構造がどう変化するかを、結節領域の数、結節線と古典運動の境界との交点数、結節領域の面積分布の指標を用いて調べた。

We investigated the transition from integrable to chaotic dynamics from the point of view of the nodal structure associated with wave functions by employing a two dimensional quartic oscillator, which has the following Hamiltonian:

$$H = \frac{1}{2}(p_x^2 + \alpha p_y^2 + x^4 + y^4) - kx^2y^2, \tag{1}$$

where the parameter k controls the dynamics of the system. Detailed studies performed at $\alpha = 1$ show that the classical dynamics of the system at $k = 0.0$ is integrable, becomes irregular as the value of k increases, and reaches an almost chaotic system at $k = 0.6$. The parameter α is introduced to break the symmetry with respect to the exchange of the x and y coordinates and is set to the value 1.01.

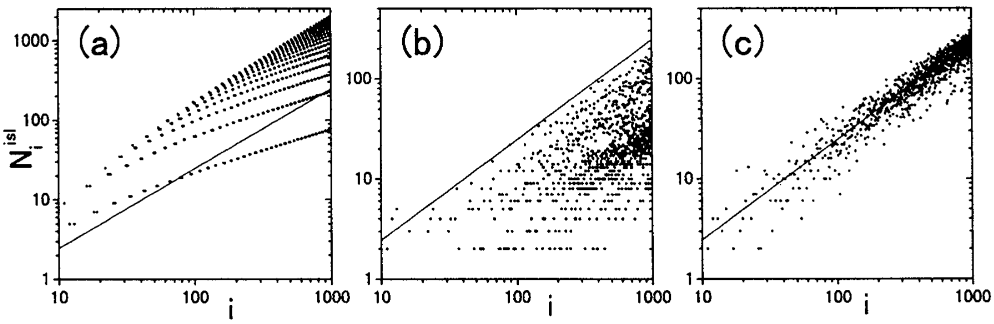

Figure 1: The number of island nodal domains for each eigenstate with the level number i: (a) $k = 0.0$, (b) $k = 0.1$, and (c) $k = 0.6$. The line shows the distribution predicted by the percolation-like model.

Figure 1 shows the distribution of the number of island nodal domains, where the island nodal domain is defined as those which do not touch the boundary of the classically allowed region. The total number of nodal domains shows a similar behavior, which, however, is influenced by the boundary, especially at the region with the small level number. Thus, the number of island nodal domains is more appropriate for a comparison with the prediction by the percolation-like model [1]. At $k = 0.0$, we find a number of regular sequences, which reflect a clear correspondence between the number of nodal domains and the definite quantum numbers for each eigenstate. When the value of k becomes nonzero, e.g., $k = 0.1$, the number of island nodal domains is drastically reduced. This is due to the transition from the crossing to the avoided crossing of nodal lines. The correlation among avoided crossings at different points is also important to reduce the number of island nodal domains. When k becomes larger, the number of island nodal domains gradually increases. Finally, at $k = 0.6$, the distribution seems to concentrate on the line predicted by the percolation-like model [1].

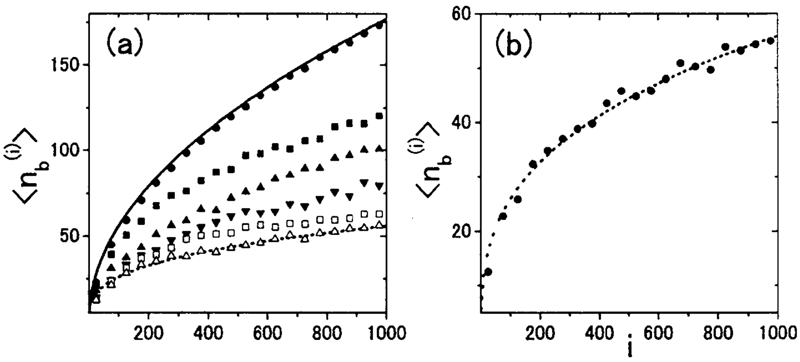

Figure 2: The number of nodal intersections with the boundary of the classically allowed region. (a) $k = 0.0$, 0.1, 0.2, 0.3, 0.4, and 0.5 from the top to the bottom. (b) $k = 0.6$. The curve in (a) shows the function $5.6\sqrt{i}$, and the dashed curve in (a) and (b) the function $5.6i^{1/3}$.

In Fig. 2, $n_b^{(i)}$, the number of intersections of nodal lines with the boundary of classically allowed region, is shown as a function of the level number i. When $k = 0.0$, $n_b^{(i)}$ is proportional to $i^{1/2}$, while, at $k = 0.6$, $n_b^{(i)}$ is proportional to $i^{1/3}$. The origin of the different i dependence is the degree of separability of the system [2].

Figure 3 shows the area distribution of island nodal domains. To measure the area s of a nodal domain, it is necessary to take into account the effect of the potential. Moreover, to obtain the area distribution, the number of nodal domains with the area s in the different eigenstate should be properly normalized [1]. The line in the figure represents the power distribution with the Fisher exponent, $n^{\mathrm{F}}(s) \propto s^{-\tau}$, where $\tau = 187/91$, which we call the Fisher line. This represents the characteristic distribution at the critical point in the percolation model [1]. The figure shows that the distribution at small values of k deviates considerably from the Fisher line, especially in the small area region. The deviation gradually diminishes as the value of k increases, and at $k = 0.6$, the distribution becomes almost aligned with the Fisher line.

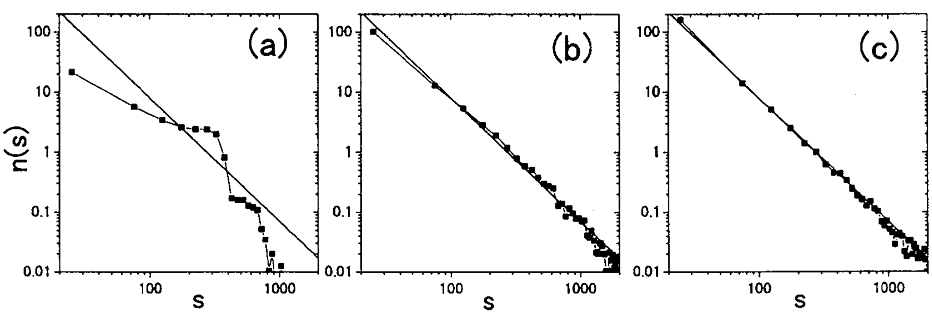

Figure 3: The area distribution of island nodal domains for (a) $k = 0.1$, (b) $k = 0.4$, and (c) $k = 0.6$. The line shows the Fisher line.

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[2] W. E. Bies and E. J. Heller, J. Phys. A **35**, 5673 (2002).